Physics 218A **Plasma Physics** **Fall 2018**

Problem Set 4: due TBA

1) (a) State and prove the two conservation equations for *momentum* in quasi-linear theory. Assume electrostatic waves.

(b) Discuss the momentum budget of electrons, ions and waves in ion acoustic turbulence.

2) Consider a current-driven system in 1-D. Electrons are Maxwellian, with centroid at u_0, temperature T_e. Ions have temperature T_i. Assume the system is collisionless.

(a) Derive an expression for the mean electric field required to maintain the electron mean velocity u_0. Your answer should depend on $\tilde{E}$ and $\tilde{f}$.

(b) Calculate the general condition for stability of this system. Do not assume $k^2\lambda_D^2 \ll 1$. Be as explicit as possible. (You can ignore the external field here.)

(c) Assuming a spectrum of unstable CDIA waves, derive an expression for the mean electric field required to maintain a stationary state. You should leave your answer as a function of u_0 and the wave spectrum.

(d) Prove that the sum of total resonant particle energy and total wave energy are conserved here, at the level of quasilinear theory. N.B. You must consider both resonant electrons and resonant ions.

3) Derive the relaxation equation for a stable plasma near equilibrium. Consider electrons with ions appearing via the dielectric. Calculate the correlator:

$$\frac{q}{m}\langle E\delta f\rangle$$

by decomposing $\delta f = f^c + \tilde{f}$, where f^c is the linear response and $\tilde{f}$ is discreteness. Use Poisson's equation to relate f^c to $\tilde{f}$, and take and treat $\langle \tilde{f}(1)\tilde{f}(2)\rangle$ as in our discussion of thermal equilibrium.

(a) Derive the expression for $\partial\langle f\rangle/\partial t$.

(b) Compare and contrast this with the Landau equation.

(c) What happens in 1-D, for an electron-ion plasma? Explain your result.

You may find it useful to consult the notes labeled “Chapter 2”, posted under Supplementary Material.

4) Read and summarize the article “Violent Relaxation” by D. Lynden–Bell, posted in Supplementary Material. Answer the following:

(a) What is the basic idea here? What does Lynden–Bell mean by “Violent Relaxation”?

(b) In simple terms, why does Lynden–Bell choose Fermi–Dirac statistics?

(c) What is the origin of irreversibility here? How is this reconciled with the fact that the Vlasov equation conserves entropy?

(d) Prove Kelvin’s Theorem for an isotropic fluid and a Vlasov plasma.

(e) What is the relation between a Jeans marginal phase space hole and a relaxed state?

5) Derive an equation for the relaxation of the mean electron distribution $\langle f\rangle$ by Coulomb collisions with electrons and ions using a Fokker–Planck approach. You may find it helpful to consult Chapter 8 of Kulsrud. In particular:

(a) Why is a Fokker–Planck approach valid?

(b) Derive Kulsrud Eqns. 18-20.

(c) Derive Kulsrud Eqns. 34, 35, 36.

(d) Derive Kulsrud Eqn. 40.

(e) Show your result is equivalent to the Landau collision integral derived in class.